General Trees

Chapter 7

Well, "non-binary" trees anyway.

General Trees

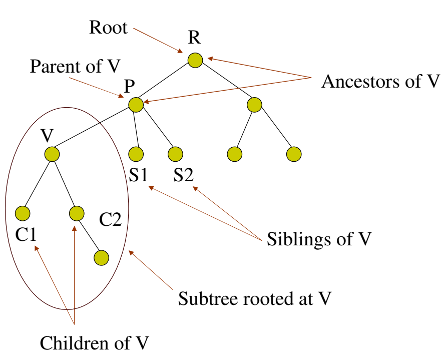
- *General trees* are similar to binary trees, except that there is *no restriction* on the number of children that any node may have.

More formally…

- A **tree**, *T*,
  - is a finite set of one or more nodes
  - such that there is one designated node *r* called the root of *T*,
  - and the remaining nodes in (*T* - {*r*}) are partitioned into $n \geq 0$ disjoint subsets T_1, T_2, …, T_k,
  - each of which is a tree,
  - and whose roots r_1, r_2, …, r_k, respectively, are children of *r*.

General Trees

- One way to implement a a general tree is to use the same node structure that is used for a *link-based binary tree*. Specifically, given a node *n*,
  - *n*'s left pointer points to its left-most child (*like* a binary tree) and,
  - *n*'s right pointer points to a linked list of nodes that are siblings of *n* (*unlike* a binary tree).



N-ary Trees

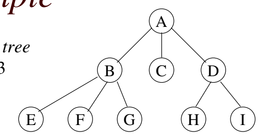
- An ***n-ary tree*** is a generalization of a binary tree, where each node can have no more than *n* children.
- Since the maximum number of children for any node is known, each parent node can *point directly* to *each of its children* -- rather than requiring a linked list.

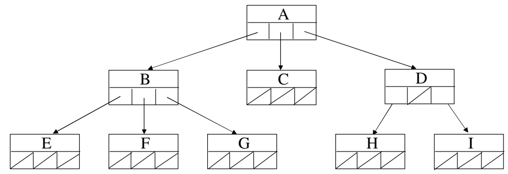
N-ary Trees

- This results in a faster search time (if you know which child you want).
- The disadvantage of this approach is that extra space reserved in each node for *n* child pointers, many of which may not be used.

N-ary Trees: *Example*

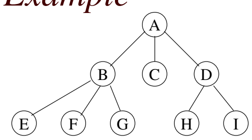
An *n-ary tree* with *n* = 3



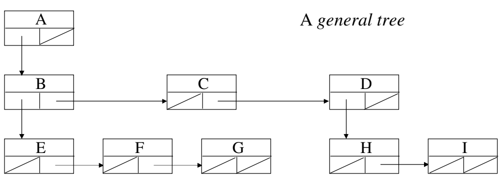


Pointer-based implementation of the *n-ary tree*

General Trees: *Example*



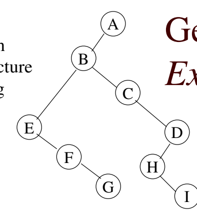
A *general tree*

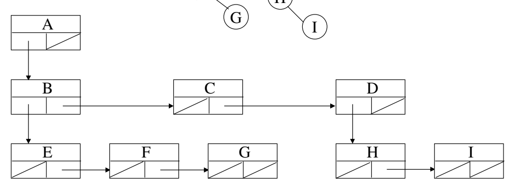


Pointer-based implementation of the *general tree*

General Trees: *Example (Cont'd.)*

Binary tree with the pointer structure of the preceding general tree





Tree ADT (Java)

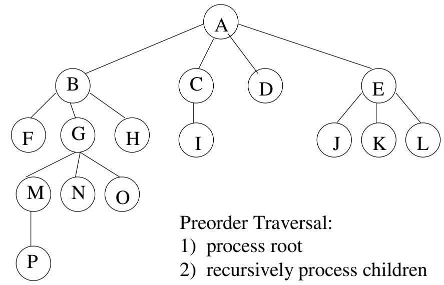
- We use positions to abstract nodes
- Generic methods:
  - integer size()
  - boolean isEmpty()
  - Iterator elements()
  - Iterator positions()
- Accessor methods:
  - position root()
  - position parent(p)
  - positionIterator children(p)
- Query methods:
  - boolean isInternal(p)
  - boolean isExternal(p)
  - boolean isRoot(p)
- Update method:
  - object replace (p, o)
- Additional update methods may be defined by data structures implementing the Tree ADT

C++ ADT

```
Class GTNode {
public:
    GTNode (const ELEM);                // constructor
    ~GTNode();                          // destructor
    ELEM value();                       // return node's value
    bool isLeaf();                      // TRUE if is a leaf
    GTNode* parent();           // return parent
    GTNode* leftmost_child();   // return first child
    GTNode* rightmost_sibling();        // return right sibling
    void setValue(ELEM);                // set node's value
    void insert_first(GTNode* n);       // insert first child
    void insert_next(GTNode* n);        // insert right sibling
    void remove_first();        // remove first child
    void remove_next();                 // remove right sibling
};
```

C++ ADT

```
Class GenTree {
public:
    Gentree();              // constructor
    ~Gentree();             // destructor
    void clear();           // free nodes
    GTNode* root();         // return root
    void newroot(ELEM, GTNode*, GTNode*);    // combine
};
```

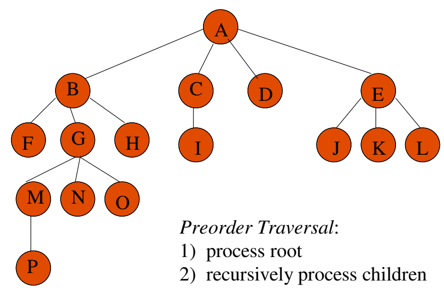


Preorder Traversal:
1) process root
2) recursively process children from left to right

General Tree Traversal

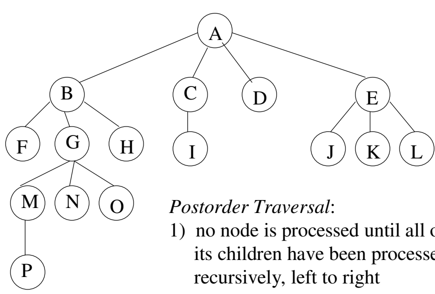
Algorithm Print (GTNode rt)   // preorder traversal from root
Input: a general tree node
Output: none – information printed to screen

```
GTNode temp
    if (rt is a leaf)
        output "Leaf: "
    else
        output "Internal: "
    output value stored in node
    temp = leftmost_child of rt
    while (temp is not NULL)
        Print (temp)      // note recursive call
        temp = right_sibling of temp
```



Preorder Traversal:
1) process root
2) recursively process children from left to right

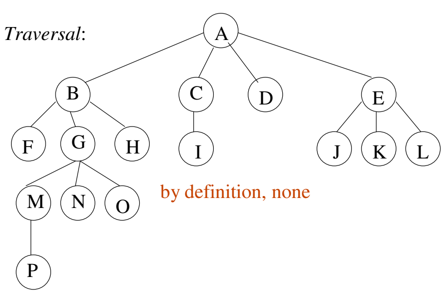
A B F G M P N O H C I D E J K L



Postorder Traversal:
1) no node is processed until all of its children have been processed, recursively, left to right
2) process root

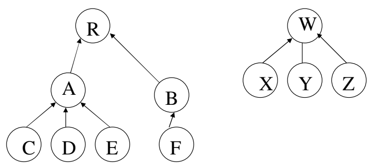
F P M N O G H B I C D J K L E A

Inorder Traversal:



by definition, none

Parent Pointer Implementation

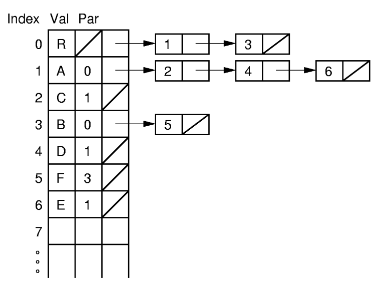


Parent's Index	/	0	0	1	1	1	2	/	7	7	7
Label	R	A	B	C	D	E	F	W	X	Y	Z
Node Index	0	1	2	3	4	5	6	7	8	9	10

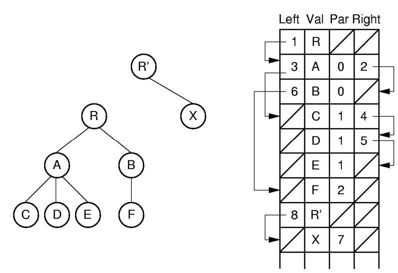
Implementations

Common ones, plus make up your own!

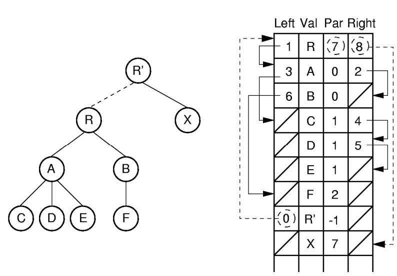
Lists of children



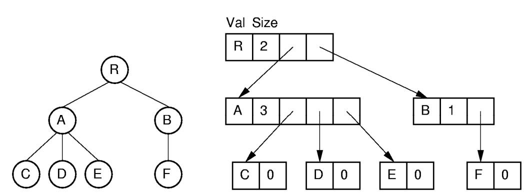
Leftmost Child/Right Sibling



Leftmost Child/Right Sibling

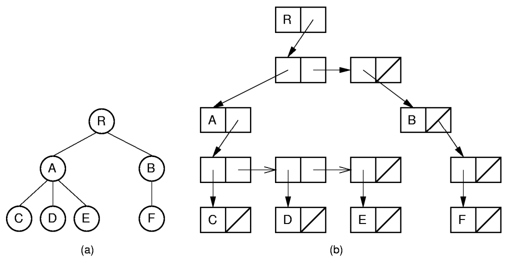


Linked Implementations



(a) (b)

Linked Implementations

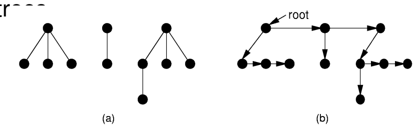


Converting to a Binary Tree

Left child/right sibling representation essentially stores a binary tree.

Use this process to convert any general tree to a binary tree.

A forest is a collection of one or more general trees.



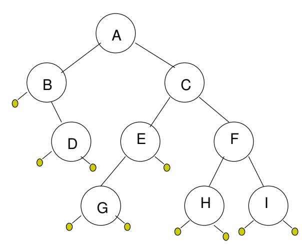
Sequential Implementations

List node values in the order they would be visited by a <u>preorder</u> traversal.

Saves space, but allows only sequential access.

Need to retain tree structure for reconstruction.

A
B
D
C
E
G
F
H
I



A
B
/
D
/
/
C
E
G
/
/
/
F
H
/
/
I
/
/

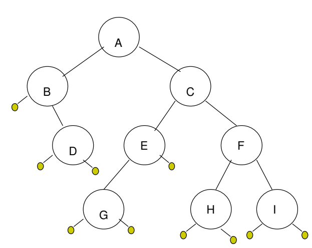
Sequential Implementations

Example: For binary trees, us a symbol to mark `null` links.

AB/D//CEG///FH//I//

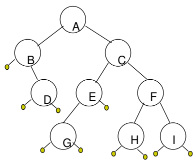
Which node was the right child of the root?

space efficient, but not time efficient



Sequential Implementations

Example: Mark nodes as leaf or internal.



A'B'/DC'E'G/F'HI

no need for null pointers when both children are null

What about general trees?

- Not only must the general tree implementation indicate whether a node is a leaf or internal node, it must also indicate how many children the node has.

What about general trees?

- Alternatively, the implementation can indicate when a node's child list has come to an end.
- Include a special mark to indicate the end of a child list.
- All leaf nodes are followed by a ")" symbol since they have no children.
- A leaf node that is also the last child for its parent would indicate this by two or more successive ")" symbols.

Sequential Implementations

Example: For general trees, mark the end of each subtree.

RAC)D)E))BF)))